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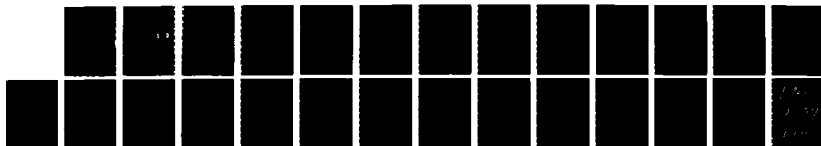
STUDIES OF SHEAR FLOWS(U) STANFORD UNIV CA DEPT OF  
MECHANICAL ENGINEERING S J KLINE ET AL 15 MAR 86  
AFOSR-TR-86-2208 F49620-84-K-0004

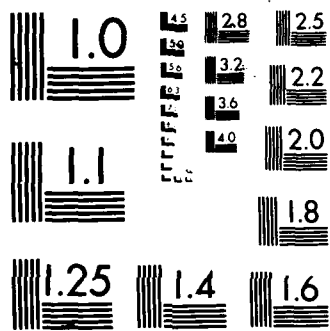
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AFOSR-TR. 86-2208

FINAL REPORT

TO

U.S. AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

ON

CONTRACT F49620-K-84-0004

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ELECTE  
DEC 23 1986  
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FROM

THERMOSCIENCES DIVISION

DEPARTMENT OF MECHANICAL ENGINEERING

STANFORD UNIVERSITY, CALIFORNIA 94305

Principal Supervisors:  
Prof. S. J. Kline  
Prof. J. H. Ferziger

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SECURITY CLASSIFICATION OF THIS PAGE

## REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS	
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT  General <sup>Approved for public release;</sup> distribution unlimited.	
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE				
4. PERFORMING ORGANIZATION REPORT NUMBER(S) Final Report Contract AF F49620-84-K-0004			5. MONITORING ORGANIZATION REPORT NUMBER(S) <b>AFOSR-TR- 86-8208</b>	
6a. NAME OF PERFORMING ORGANIZATION Stanford University	6d. OFFICE SYMBOL (If applicable) Mech. Engrg.	7a. NAME OF MONITORING ORGANIZATION <del>USAF AFOSR</del> AFOSR/NA		
6c. ADDRESS (City, State and ZIP Code) Stanford, CA 94305-3030		7b. ADDRESS (City, State and ZIP Code) Building 410 Bolling AFB, DC 20332-6448		
8a. NAME OF FUNDING/SPONSORING ORGANIZATION <del>USAF AFOSR</del> AFOSR/NA	8d. OFFICE SYMBOL (If applicable) N/A	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER <b>F49620-84-K-0004</b>		
8c. ADDRESS (City, State and ZIP Code) Building 410 Bolling AFB, DC 20332-6448		10. SOURCE OF FUNDING NOS.		
11. TITLE (Include Security Classification) Studies of Shear Flows		PROGRAM ELEMENT NO. <b>61102F</b>	PROJECT NO. <b>2307</b>	TASK NO. <b>A/4</b>
12. PERSONAL AUTHOR(S) S. J. Kline, J. P. Johnston, <u>R. J. Moffat</u>				
13a. TYPE OF REPORT Final	13b. TIME COVERED FROM <u>1/84</u> TO <u>12/85</u>	14. DATE OF REPORT (Yr., Mo., Day) 85-03-15		15. PAGE COUNT 24 (2 parts)
16. SUPPLEMENTARY NOTATION				
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary; and identify by block number)	
FIELD	GROUP	SUB GR		
19. ABSTRACT (Continue on reverse if necessary and identify by block number)				
<p><b>TASK A:</b> Work covered three projects all concerning "zonal modeling" of turbulent flows: Free shear layers, boundary layers, and complex flow fields with separation.</p> <p>Work on the class of free shear layers was completed except for preparation of a report describing the results. The effort has been very successful. A fast running model has been created that predicts all cases of free shear flows where sufficient data allow checking to 10%, roughly as accurate as the data. Included are: jets, wakes, and mixing layers for planar and axisymmetric cases covering both near field and far field and co-flowing cases where data exist. In short, all the existing classical cases. This is done with a two parameter model. Adjustments for transitions from one zone to another within this class are all successfully achieved using a first order linear ordinary differential equation, leading to exponential readjustments. Because of the processes used in the zonal modeling, significant new</p>				
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22a. NAME OF RESPONSIBLE INDIVIDUAL <b>DR JAMES D WILSON</b>		22b. TELEPHONE NUMBER (Include Area Code) <b>202- 767-4935</b>	22c. OFFICE SYMBOL <b>AFOSR/NA</b>	

information about the necessary correlating governing parameters for these flows and what they represent in the physics has been realized. Details will be provided in a report scheduled for late 1986.

Work on a zonal model for boundary layers is well advanced. Zero pressure gradient and favorable pressure gradient cases have been modeled. Work during 1986 will concentrate on adverse pressure gradients and effects of wall curvature (both convex and concave).

Modeling of complex fields, using the backward-facing step as a proto-type, is well advanced. The reasons for mediocre to poor performance of all two-equation models in the 1980-81 AFOSR-Stanford Conference on this problem have been located. They lie half in inadequate means for checking numerical accuracy, and half in the failure to account for the effect of streamline curvature on the turbulence production and turbulent shear. As in the case of the free shear layers, the zonal modeling not only succeeds, but the processes used also illuminate the necessary parametrizations and the underlying physical effects the parameters model. This is quite distinct from conventional 2-equation models where constants are set beforehand, and relatively little is learned except how well, or badly, the model works. The ability to provide good estimates of numerical accuracy at all grid points, developed in related work in this division, has been critical in this work.

Good estimates of the flow field up to reattachment have been achieved. However minor inaccuracies occur after reattachment owing to the fact that standard 2-equation models fail to predict accurately the reattached boundary layer and its readjustment downstream to an equilibrium layer. It will be necessary therefore to create a zonal model for reattached layers and their readjustment. This result is not surprising in view of the data by J. Kim of this laboratory, and that of Bradshaw and Wong on this flow situation. Work during 1986 will complete a model for the reattached layer and study cases where the flow channel is turned to provide parametric testing over a range of conditions.

**TASK B:** This task was a study of the structure of a turbulent boundary layer on a wall with concave curvature in the longitudinal direction ( $R_{wall}/\delta \approx 136$ ), and to investigate the effects of concave curvature on convective heat transfer. The work was conducted in a large scale, low speed water channel, in a zero pressure gradient, turbulent boundary layer of  $Re(\theta) \approx 1400$ . Flow was visualized by wall-slot dye injection of dyes (vegetable and Fluorescene) into the wall layers. Structure was observed and filmed using back-lighting, and the laser light sheet method. Quantitative U- and V- component mean velocity and velocity fluctuation profiles were obtained simultaneously by laser velocimetry, down to y-plus value of 2 to 7. Turbulent "streak" bursting rates and spectra were obtained. Surface heat transfer was visualized and quantitatively measured using liquid crystal films on the test surface. A method for digital processing of video images from the liquid crystal surface was developed, using "hue" (i.e. color) processing rather than monochrome intensity processing.

Concave curvature has a destabilizing effect on turbulent shear layer structure, seen here as a strong amplification of the negatively correlated motions normal to the wall. The visualized structure in the curved layer shows effects not seen over flat walls, inflows and outflows, of large-scale (order  $\delta$  in span and two  $\delta$  long). These changes increase the mean skin friction coefficient by 40% and the Stanton number by 20%. The structure of the wall layer turbulence, below y-plus  $\approx 100$  is unchanged by curvature. Cross stream mixing is much stronger in the middle and outer parts of the layer where the low frequency, large-scale motions are important. No stationary, spanwise variations, similar to Taylor-Gortler vortices are seen unless some upstream disturbances (wakes of vortex generators, dirt in plenum screens, etc.) are present to lock the large-scale inflows and outflows in position.

I.

INTRODUCTION

A. Objectives of the Work

The objective of the work reported is construction of zonal models for accurate prediction of turbulent flows in rapid-running computer programs.

The work follows the ideas set down in the discussion of zonal modeling by S. J. Kline in Vol. II of the Proceedings of the 1980-81 AFOSR-Stanford Conference on Complex Turbulent Flows. This discussion noted that the fast-running models available lack sufficient "span" to predict all classes of turbulent flows of engineering importance in a standard, invariant form. It therefore suggested that the models be treated as zonal. In the zonal approach, the constants in the models are adjusted for each important zone of the flow, where the word zone implies a region with a particular type of flow physics. (K)

In the proposal for the work, it was also noted that the parametrization of the flows and the selection of appropriate constants would ultimately need to be guided by the domain over which accurate results could be obtained, and that might well be different from pre-conceptions embodied in conventional taxonomies of the flows.

B. Initial Program

The initial program included four types of flows:

1. Homogeneous turbulence
2. Free Shear Layers
3. Boundary Layers
4. Complex Fields--using the family of backstep flows as exemplary

Details of progress on each of these phases of work are given in section II of this report.

C. Contract and Report Status

This is a final report on AFOSR Contract F49620-K-84-0004. However, the work is not yet complete and is being continued under a successor contract F49620-K-86-0008.

Two phases of the initial four phases of the work are complete. These cover homogeneous turbulence and free shear layers. A final detailed report on these two phases is being prepared and will be submitted in the summer of 1986.



DATE	INITIALS
A-1	

#### D. Results obtained

The results obtained thus far are excellent. They fulfill, very successfully, the program goals for homogeneous flows and free shear layers. Results thus far for the boundary layer and the fully-separated flow case appear to follow similar lines.

As explained more fully in the letter transmitting this report to Dr. James Wilson of AFOSR, these results emerge very much along the lines predicted in the proposal and in the discussion on zonal modeling cited above.

## II.

### RESULTS

#### A. Homogeneous flows; and Free shear flows.

Research Assistant: K. L. (Steve) Tzuoo

Academic year 1984-1985:

The zonal modeling of homogeneous flows was finished and a complete report written. Initially a computer code was written to solve the equations for thin shear layer type flows using Keller Box method. A laminar jet was used to test the code and the result was satisfactory except it tended to give oscillations in the solutions if the initial conditions given were not perfect. Several modifications of the code were made, but the problem with initial conditions persisted. Finally a Crank-Nicolson method was used with non-uniform staggered expanding grid. This numerical scheme is identical to that used by Mr. S. Bordalo who is working on the boundary layer flows. With this numerical tool, the solutions behave smoothly.

The code was then implemented to solve turbulent flows with the ability of solving the K-e equations simultaneously with the mean flow. A "pure" jet was first tested using the standard K-e model (without changing the standard model constants). The results showed excellent agreement with the experimental data. Both global behavior and the detailed profiles agree with the experiments within the experimental uncertainty. During this period, a literature survey of experiments on all the basic free shear flows was also made to help understand the differences in physics of various cases of free shear layers, and to determine which cases were accurate enough and supplied enough initial data so that they were adequate for verifying model performance.

The study of the physics of the flows was focused on what can be called "structure functions". These are detailed plots of the primary terms of the Reynolds stress matrix for the given flow. This provides a method for determining if various flow cases can be represented using the same values of constants, or on the contrary will require different values of constants in the K-e equations in order to obtain accurate output.

Academic year 1985-1986:

Standard K-e model was tested in co-flowing jets, wakes, and mixing layers. The predictions of the standard model were not adequate for these flows. For example, the spreading rate of the plane far-wake predicted by the standard k-e model is about 35% lower than the experiments, and the spreading rate of an axisymmetric jet is overpredicted by more than 30%. A careful study of the physics using the method of structure functions described above showed that:

1. As flows with free stream velocities develop downstream, the transverse velocity becomes smaller and therefore the streamlines become more parallel. Owing to lower entrainment, the eddies penetrate less into the inertial stream and the spreading rate becomes smaller. In a situation where the streamlines are nearly parallel, it therefore appears that the transverse diffusion processes dominate the turbulence transport. From the various profiles that the standard K-e model predicted, it was concluded that the diffusion processes described by the standard K-e model are not strong enough under these conditions.

2. For rapidly-spreading axisymmetric flows, the shear stress and turbulent kinetic energy correlation level becomes smaller due to vortex stretching. For example, for axisymmetric "pure" jet,  $\langle u'v' \rangle / K$  is only about 0.22 compared to about 0.3 in all the plane flows.

As a result of 1 above, it is concluded, from zonal modeling point of view, that there are two extremes in the free shear flows; one being when the ratio of the free stream velocity and the maximum velocity difference in the layer is zero, and the other when this ratio is infinity. Since the standard k-e model works well for the first extreme as described above, the remaining task is to model the other extreme. This was done by fitting the far-field plane wake flows. There are two model constants that need to be changed to reflect the fact that the diffusion processes are enhanced in this extreme.

The region in between the two extremes can then be considered as a zone of readjustment. The same exponential form of sliding as that used for the homogeneous flows (first-order lag equation) was tried for this readjustment and found to work satisfactorily.

This was fitted using wake data. This provides a complete model for the plane free shear flows that covers the entire range of the velocity ratios. This model was used to test the plane co-flowing jets and mixing layers; the results were very satisfactory. All cases known to be of adequate accuracy and supplying enough data for testing check within plus or minus ten percent for velocity profile, spreading rate and shear stress. One constant is required to allow for the physics of the readjustment process.



In axisymmetric flows, the same changes in the two model constants were found to be adequate. However, one additional matter must be taken into account to provide results for all cases. In particular, point 2 indicates the need for the model to include the effect of vortex stretching in the early part of the flow field. A vortex stretching parameter was introduced and used as a modifying factor for the constant  $C_u$ . This modifying factor does not affect results for the plane flows, but does decrease the value of  $C_u$  in axisymmetric flows when the spread is rapid as suggested in point 2. With addition to the model of this modifying factor, the axisymmetric co-flowing jets and wakes were tested and the results were again all within plus or minus ten percent of the data. Since ten percent scatter is typical of even the best turbulent flow data when the variations between laboratories is considered, and no fundamental solutions for turbulent free shear flows are available, this is as good an accuracy as can be obtained from modeling.

It is important to note in addition that this solution for the class of free shear flows is not only more accurate than other available models but also more complete. (Other solutions do not predict the nearfield). Hence this extension in completeness is significant for design purposes.

A good overall idea of the improvement in accuracy obtained by using a zonal model, in contrast with an assumedly universal 2-equation model can be seen in the following table. Showing the distribution of accuracies for all adequate cases we know for homogeneous flows and free shear flows respectively.

Entries in the table represent the percentage of flow cases accurate to the limits shown or better.

#### Homogeneous Flows

Worst point on curve agrees with data within	<10%	<25%	<50%	>50%
Present Zonal Model	86	10	4	0
Standard K-e Model	16	16	17	51

#### Free Shear Flows

Worst point on curve agrees with data within	<10%	<25%	<50%	>50%
Present Zonal Model	100	0	0	0
Standard K-e Model	25	44	12	19

In reading Table I, several things are worth note. First, the free shear layer flows are more important in engineering applications so that the accuracy in free shear layer cases is primary in these results. Second, the accuracy of the K-e methods is not subject to prediction; that is, one does not know typically where a given application will lie in terms of accuracy. Hence the 31% of case that are less than 25% accurate is a serious drawback of the method. Third, the homogeneous flows are more difficult to predict than the free shear flows, contrary to what might be imagined. Finally, the standard K-e methods was not intended to do homogeneous flows.

The results of the work thus provide a very satisfactory model for the whole class of free shear flows by the zonal approach. Computation times on a VAX 750 are the order of minutes or less, and hence computing times are not a problem for these flows, and enough freedom is left to consider constructing more complex flow fields using this model to represent free shear layers that appear in such flow fields.

In sum, the present representation shows that the entire class of free shear flows that are now well documented can be represented by zonal models with two free parameters, one major and one minor and the use of a first-order lag equation for the readjustment over the total physical range of these parameters. The major parameter is the ratio of the free stream velocity to the maximum velocity difference in the layer. The minor parameter represents the effect of vortex stretching of the principal flow structure in axisymmetric flows when that effect becomes large which occurs primarily in the near field of axisymmetric flows.

These results do not encompass all the known physical effects that occur in common complex flow fields as discussed in section C below. However, they do provide a method for deciding how to parametrize and model flows in order to achieve success and they do provide an important unit of input for the zonal modeling of complex flow

Thus, the zonal method has been shown to be completely successful for all known adequately documented cases of homogeneous flows and free shear flows. The modeling necessary to achieve these results also enlarges our understanding of the flow physics and how it can be parametrized to provide generalized representations.

## B. Boundary Layer Flows

Research Assistant: Sergio Bordalo

The objective of this phase of work is the development of turbulence models within the framework of ZONAL MODELING for computing attached boundary layer flows including the effect of

pressure gradient, blowing-suction, and curvature.

An algorithm has been produced to solve the parabolic partial differential equations of the thin boundary layer. It involves the use of a non-uniform adaptive staggered grid with variable step size applied to the domain of transformed variables. A finite difference method is employed to obtain the velocity profiles.

The Keller Box method was tested for laminar flows with zero, favorable and mild adverse pressure gradient, and turbulent flow with zero pressure gradient using a mixing-length formulation. The results were unsatisfactory since excessive sensitivity to initial conditions could not be eliminated. A Crank-Nicolson method was tested for the same conditions and yielded much better output. These results are the same as in Phase A of the work so that both phases have settled on the same numerical method.

The next step consists of the development of zonal models based on the "K-e model". The standard "K-e model" was tested for zero-pressure gradient case and found to be satisfactory. The model is now being tested for favorable-pressure gradient. The next tests will cover adverse-pressure gradient, curvature, blowing/suction.

#### C. Complex Flow Fields--Using the Single Backward-Facing Step as Exemplary

Research Assistant: Ram Krishna Avva

Work Done

Academic year 1984-85:

The single backward-facing step was chosen as an exemplary case for a number of reasons. It is a complete flow field embodying recirculation, separation and reattachment, free shear-layers, attached boundary layers, potential flow regions, and the effects of curvature and reattachment on shear layers. It thus incorporates most of the common zones of incompressible flow fields and many of the total set of physical effects that must be modeled to provide a complete zonal model (Or any other kind of complete model of complex flow fields). It was used as a "core case" in the 1980-81 AFOSR-Stanford Conference on Complex Turbulent flows. It is about at the cutting edge of research in CFD of viscous flow fields at this time. As part of the work of the 1980-81 AFOSR-Stanford Conference, we created several extensions of the basic data involving systematic variation in the downstream angle of the channel relative to the incoming flow direction. Thus solutions created for the base case can be checked against a systematic parametric variation in the flow field geometry, providing a check on generality of modeling not available for other well documented cases.

A Numerical code was developed to solve two dimensional Navier Stokes equations. The code uses the SIMPLER method [1] and is capable of solving any incompressible elliptic flow. It was validated by solving a number of laminar flows including the 2 dimensional shear driven cavity and 2 dimensional single backward-facing step. The standard K-e model was then incorporated to predict turbulent flows. In these equations, the Reynolds averaged NS equations are solved along with transport equations for kinetic energy (K) and dissipation of (e) of the turbulent motions in an iterative manner. Wall functions are used in the near wall regions. See for example Rodi [2].

Academic year 1985-86 :

The backstep data of Kim, et al [3] (The standard case from the 1980-81 AFOSR-Stanford Conference) was used as a first case. The flow had a 2:3 expansion. For this case, standard K-e model predicts the overall pressure recovery quite well as any model will do that incorporates an appropriate momentum equation since that result depends only on overall momentum conservation.

However, the standard K-e model fails to predict the velocity and turbulent profiles both in the recirculating and recovery zones. This is true for all K-e models presented in the 1980-81 AFOSR-Stanford Conference. These models also underpredict the recovery of the turbulent energy and Reynolds shear stress in the recovery zone. And, they underestimate the length of flow to the mean reattachment line, the key geometric parameter of the flow, by about 20%.

In examining the reasons for these inadequacies of the existing two equations models, five possible sources of errors have been isolated: (i) inadequate numerical accuracy in critical regions, particularly in the region just behind the sharp step; (ii) lack of representation in the model of the physical effects of curvature, that is the centrifugal force field, on the turbulent stresses; (iii) lack of adequate modeling of the return flow in the recirculating zone and hence possibly of its effect in turning the shear layer just after it separates from the step owing to transverse momentum; (iv) inadequate accuracy of the initial conditions at the step owing to lack of data, or of computation of the channel flow sufficiently far upstream; (v) inadequate modeling of the zone of reattachment and also the recirculation zones.

Since all these problems will recur frequently in modeling complex flow fields, it becomes important to accumulate understanding on what is needed for adequate modeling in each case and which affects are important under what circumstances. Considerable information has been accumulated, via work thus far on the first four of these potential sources of modeling difficulties. It appears that all of the first three are important and must be accounted for. Item (iv) appears to be unimportant for the single backward-facing step. Since it will be relatively large

in this case, compared to the common instances where such a problem occurs, this is probably adequate information about this potential difficulty. Items (i) and part of (ii) are matters of doing good computations numerically and being careful. They can be handled using the estimates of numerical accuracy being created in other parts of the Stanford researches on turbulent flows and in other research groups. Item (iii) is an important physical effect in many cases, and appears to be in the backward step as well. The free shear layer, in the backward step initially is essentially straight, but as it approaches reattachment, considerable curvature of the streamlines occurs. The direction of this curvature is such that it will reduce the level of turbulent shear stresses. This effect would then tend to make the reattachment length longer than would be predicted by standard models which do not include terms to account for curvature effect. That direction agrees with the errors found in the standard model in the 1980-81 AFOSR-Stanford Conference.

More detailed comments on each of these effects follow.

Grid refinement studies were carried out and it was found that the solution improved considerably with refinement. A reattachment length of over six step heights was obtained. This is significantly higher than the value obtained by most computers in the 80-81 conference. However this value is still smaller than the experimental value of seven step heights.

The effect of resolving the discontinuity at the step was studied, first by adding an upstream channel and secondly by clustering the nodes near the step. The main effect of this was reflected in improved pressure profiles. The reattachment length and the velocity and turbulence profiles were not significantly affected.

Experiments done on curved shear layers have demonstrated that the turbulent shear level is strongly affected by curvature. The detached shear layer in the backstep flow is noticeably curved upstream of the reattachment point. However the standard K-e model does not have any curvature sensitive terms and so it is not too surprising that the standard model fails to predict the reattachment length correctly. The modified K-e model of Bardina & Ferziger [5] was used as it accounts for the effect of rotation. However it did not have a positive effect on the prediction. A modified model suggested by Leschziner & Rodi [6] was used and preliminary tests show a significant increase in the reattachment length. This model, which was derived from Gibson's Reynolds stress closure model [7], uses an expression for  $C_{mu}$  such that it is curvature dependent.

#### FUTURE WORK

In the coming year, the experimental observations made by Eric Adams [8] and John Vogel [9] in developing some zonal models. In particular, the boundary layer beneath the recirculating bubble seems to be quite different from an ordinary

wall bound turbulent boundary layer and so needs special attention to predict the recirculating zone accurately. The importance of this fact on the overall flow field is not yet known, but needs investigation. The recovery zone, which is a non-equilibrium boundary layer, also needs a model different from that of the standard  $K-\epsilon$  model as the latter is observed to be inadequate for non-equilibrium boundary layers. A model of this was provided by Kim et al [3], but needs to be generalized and brought into the framework of this zonal modeling program.

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FINAL REPORT

AFOSR-F49620-84-K-0004

TASK B

For period  
1 January 1984 -  
31 December 1986

J. P. Johnston, R. J. Moffat

Department of Mechanical Engineering

Stanford University

Stanford, CA 94305

30 June 1986

RJM:Misc.



## I. General Introduction

This is the annual report for contract AFOSR #F49620-84-K-0004, covering "Theoretical and Empirical Studies of the Basic Structure of Turbulent Shear Flows, Including Separated Flows and Effects of Wall Curvature." The period covered is Jan. 1 through Dec. 31, 1985.

This contract includes work on two distinct projects.

Task A. Construction of zonal models for computation of complex turbulent flows, which was reported separately by S. J. Kline and J. H. Ferziger.

Task B. Study of turbulence structure and heat convection in turbulent boundary layers on concave surfaces, which is reported here.

The present report covers Task B.

## TASK B

THE TURBULENCE STRUCTURE AND CONVECTIVE HEAT TRANSFER  
IN A TURBULENT BOUNDARY LAYER ON A CONCAVE SURFACE1. Goals1.1. Introduction

Kreith\* first showed, in 1955, that concave curvature increased heat transfer, although the fact that curvature affected the turbulence structure had been known since the early 30's. Between 1955 and 1967, there does not appear to have been much activity. In 1967 Schneider and Wade measured heat transfer in a curved-duct flow, and in 1968 Thomann made local measurements of boundary-layer heat transfer in a supersonic flow with convex and concave curvature. In every instance, concave curvature resulted in an increase in heat transfer. The increase was assumed by many to be the direct consequence of streamwise vortices within the boundary layer, caused by a Taylor Gortler instability, but there was dissent. Eskinase and Yeh (1956) reported an increase in heat transfer, but no evidence of streamwise vortices. The issue has become increasingly important as aircraft engine designers have pressed closer and closer to the limits of assurable prediction of heat transfer. It is desirable to be able to predict the heat-transfer coefficient within 5% on a turbine blade, yet this cannot be done at the present state of the art. Most of the current prediction programs for boundary-layer heat transfer are two-dimensional. If Taylor-Görtler vortices are important in the concave-wall boundary layer, then three-dimensional codes will have to be developed. On the other hand, if the increase in heat transfer is a result of a generally increased turbulence activity, but still two-dimensional, then existing codes can be simply modified to acknowledge the curvature effect, and no major changes in computational philosophy need be undertaken.

The objective of this work is to identify the mechanism whereby concave curvature increases the heat transfer through a turbulent boundary layer. This involves careful documentation of the fluid mechanics and of the heat transfer. It is necessary to establish a well-qualified flow on a concave surface, demonstrate that the heat transfer is increased, and then determine the fluid-mechanic and thermal behavior of the boundary layer carefully enough to establish whether or not streamwise vortices played an important role. As in most convective heat-transfer problems, the fluid mechanics must be thoroughly understood before the heat-transfer study can be begun.

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\*References listed here are in the report HMT-35, listed at the end of this section.

## 1.2 Background and Objectives of the Fluid-Mechanics Study

Concave curvature has a relatively large, unpredictable effect on turbulent boundary layers. Past studies of turbulent boundary layers on concave walls have emphasized quantitative, single-point measurements. For example, skin-friction and turbulence levels are shown to increase when the boundary layer goes from a flat wall to a concave wall. However, there is disagreement over the cause of these results. Some recent studies have reported large-scale, spanwise variations in mean velocity and skin friction to support the idea that an array of large-scale, counter-rotating vortices exists within the concave turbulent boundary layer--structures similar to the Taylor-Görtler vortices seen in laminar boundary layers on concave walls. However, in other studies these large-scale variations were not found. Even among those who did find stationary, large-scale variations, there are inconsistencies in the relative size and spacing of the variations compared with the boundary layer thickness and the radius of curvature, the most reasonable length scales. Furthermore, the effects of concave curvature on the basic elements of the boundary-layer structure (e.g., streaks and bursts) have not been studied, and at the start of the original study, no adequate picture of the overall flow field existed.

The general goal of this study is to obtain improved qualitative and quantitative understanding of concave turbulent boundary layer flow. The knowledge gained should permit development of more realistic computational models of the fluid dynamics for use at all levels, from integral equations up to the full Reynolds stress equations.

The findings and accomplishments of the first phase of the fluid dynamics studies are summarized in the final report (May, 1984) on the original contract, AF-0010. Complete technical details are given in the three references by Jeans and Johnston (1982, 1983, and film supplement to Report MD-40).

In summary, a new channel-flow facility for the study of concave turbulent boundary-layer flow was designed and built. Studies of the flow were carried out using flow visualization (dye injection into the wall layers and hydrogen bubbles). In addition, the hot-film velocimetry method was employed to obtain profiles of mean velocity,  $U(y)$ , and profiles of fluctuation,  $u'(y)$ , about the mean. The fluid is water at a mean speed of 15 cm/sec. The flow develops over a long (4.9 m) flat surface before it enters a 90° concave bend (wall radius = 134 cm). The opposite wall of the channel is contoured to cause the static pressure along the test surface to be constant. At the start of the bend, the boundary layers are 8 cm thick and fully turbulent ( $U\theta/\nu = 1300$  to 1400). The flow has good spanwise two-dimensionality.

In the studies of Jeans and Johnston, it was shown that the basic instability mechanism that creates laminar Görtler cells also acts in the turbulent boundary layer over a concave surface. However, the expected stable laminar pattern of longitudinal vortices was not seen. Rather, a pattern of large-scale structures, now referred to as roll-cells, appears in the concave region of the test surface. These cells are neither stationary in space or time, and their longitudinal extent is only two to four times their length scales in the  $y$  and  $z$  directions. The latter scale is of the order of the boundary-layer thickness, much larger than the largest eddy sizes in the turbulence. The quantitative data showed that the development of the roll-cell structure

had profound effects on the mean-velocity profiles and the turbulent fluctuations. However, the quality of the  $U$  and  $u'$  data was not satisfactory for further detailed study of the flow.

The objectives of the current program, under AF-0004, were formulated in 1982, and we started to carry them out in 1983. Much progress (outlined below) was made in the two years of the contract. Our objectives were to:

- (i) install and modify, as required, the two-component ( $u$  and  $v$ ) LDV system on the concave wall facility. This has been accomplished and excellent data obtained.
- (ii) investigate means for stabilizing the spanwise locations of the longitudinal roll cells so that detailed studies of their turbulence structure might be accomplished. This is completed and reported in Barlow and Johnston (1985). Carefully sized and placed vortex generators were used for roll-cell stabilization.
- (iii) Study, in all feasible ways, the quantitative features of concave-wall, turbulent flow structure. In particular, we want to find the mechanisms by which the large-scale features, the roll cells, affect the structure of a turbulent layer so that more rational approaches to its modeling, at practical and fundamental levels, may be made.

### 1.3 Objectives of the Heat-Transfer Study

The primary objective of the heat-transfer study was to determine whether or not streamwise vortices (that is, Taylor-Görtler vortices) played a significant role in raising the heat-transfer rate on a concave wall. The concept of a streamwise vortex involves an organized motion of the fluid over a long streamwise distance--such structures cannot be detected at any single point. Furthermore, there is no assurance that such structures, if present, would be stationary--they might meander. The measurement problem was thus expressed in the following form: devise a way to find out if the increase in heat transfer is due to large-scale structures within the boundary layer, whether they are stationary or meandering.

The requirement for "full-field" knowledge suggested that the best approach would be a visualization technique, applied over the entire surface, which would make visible the heat-transfer coefficient distribution everywhere at the same instant. Such a technique had already been developed, at Stanford, for steady-state use in air. Application to transient studies in water seemed feasible, and this approach was selected for the study.

The overall program then consisted of the following steps:

1. Develop a curved-wall tunnel which produces a flat-plate boundary layer with normal characteristics at the entrance to a concave test section.
2. Qualify the visualization method for accurate measurement of mean heat transfer on a flat wall in water.

3. Qualify the visualization method for unsteady heat transfer at frequencies as high as might be expected of a meandering vortex.

4. Demonstrate that the average heat transfer on the concave wall is larger than on the flat wall by the expected amount (20-30%), all other factors remaining fixed.

5. Photograph, interpret and examine the wall-visualized distribution of heat-transfer coefficient, looking for evidence of streamwise structures.

## 2. Accomplishments

To preserve the structure thus far established in this report, the accomplishment of the fluid-mechanic and heat-transfer studies are reported separately. This should not be construed to mean that the two programs were run independently--far from it. Professors Johnston and Moffat worked closely together in the planning, execution, and interpretation of the results, as did the research assistants.

### 2.1 Fluid Mechanics

Details of the first phases of work are contained in the 1982 publications by Jeans and Johnston. The work under this contract is reported in the papers by Barlow and Johnston.

Specifically, the accomplishments of the contract period were:

(i) We completed the installation and modifications to the laser-velocimeter system so that we are able to measure instantaneous  $u$  and  $v$  velocity components. At any particular streamwise ( $x$ ) station, we can control the traversing unit to produce profiles normal to the wall,  $y$ -direction, or spanwise to the wall,  $z$ -direction, over a distance of  $\pm 20$  cm with respect to the centerline of the flow. Our current data on  $u$  appears to be accurate down to  $y^+$  values of 2 to 3, and  $v$  is accurately measured from  $y^+ = 6$  to 7 out to the edge of the boundary layer.

(ii) A semi-automated data-acquisition system is in place.  $u$ ,  $v$ , and probe position are continuously recorded on our laboratory VAX and the records used for detailed analysis. For a single location in the flow, data-acquisition rates are fast enough, and acquisition times long enough (2 to 5 min.) to allow frequency analysis over the full range of interest; over a range of periods from about 200 sec down to 1/50th of a sec. the latter corresponds to a time scale shorter than viscous decay scale of the smallest eddies, and the former is typical of the roll-cell lifetime.

(iii) We have obtained a full set of data for cases with and without roll cells. These include complete  $u$ ,  $u'$ ,  $v'$ , and  $\langle u'v' \rangle$  profiles at the upstream flat station, and at 15, 30, and 60° of turn in the curved region. When the vortex generators were in place, complete profiles were taken in both inflow and outflow regions, between two adjacent roll cells (see Barlow and Johnston, 1985).

(iv) We developed software that enables us to use the raw  $u'$  and  $v'$  data records to detect sweeps and bursts at various  $y^+$  locations. The uv quadrature method of Bogard and Tiedermann works well for this purpose. The VITA method was also attempted. Both were checked against visual observations of the burst frequency in the flat-wall part of the flow. The Tiederman method is to be preferred.

(v) A method for extracting spectra from the  $u'$ ,  $v'$ , and  $\langle u'v' \rangle$  data was developed. Studies of these spectra were reported at the 5th Symposium on Turbulent Shear Flow, Cornell, Aug. 1985. Details of these studies and those of item (iv) are given in our major technical report, MD-47.

(vi) The primary technical findings for the contract period are:

(a) The major effect of concave curvature is to amplify large-scale, negatively correlated motions normal to the wall. Mixing is enhanced, and high-momentum fluid comes closer to the wall, causing a fuller velocity profile and a higher wall shear.  $C_f$  at the  $60^\circ$  station is about 40% higher than the value expected in a normal flat boundary layer at the same momentum-thickness Reynolds number.

(b) Concave curvature causes large increases in turbulence intensities across the middle and outer parts of the boundary layer, especially for fluctuations normal to the wall. Comparing the flat and  $60^\circ$  boundary layers, the peak value of  $v'U_{pw}$  increases by 50%, the ratio of Reynolds normal stresses,  $\langle v^2 \rangle / \langle u^2 \rangle$ , increases by 80%, and the correlation coefficient,  $R_{uv}$ , increases by 40%. Spectral analysis shows that these increases can be attributed to increased energy in low-frequency, large-scale fluctuations.

(c) Concave curvature has smaller effects on structure near the wall. The friction velocity,  $u_\tau$ , increases significantly. However, when the local value of  $u_\tau$  is used for scaling, data for turbulence intensities and shear stress collapse to universal profiles inside  $Y^+ = 15$ .

(d) The response of a turbulent boundary layer to the onset of concave curvature involves two overlapping stages.  $R_{uv}$  and the shear-stress parameter,  $a_1$ , show significant increases within five initial boundary-layer thicknesses,  $\delta_o$ , of the start of curvature, indicating a rapid initial reorientation of existing eddies. This is followed by a slower transfer of energy from the mean flow into large-scale motions normal to the wall.

(e) The visualized structure of the concave boundary layer is dominated by large-scale inflows and outflows. Inflows cause sublayer streaks to diverge, spreading sublayer fluid laterally toward neighboring outflows. Most of the fluid that leaves the sublayer does so in the large-scale outflows. Large-scale inflows suppress turbulence and bursting, while large-scale outflows enhance both. Consequently, the spanwise variation of skin friction due to the inflows and outflows is less than one might expect, given the strong influence of these large-scale structures on the visualized, near-wall flow.

## 2.2 Heat Transfer

The heat-transfer coefficient distribution has been made visible on a concavely curved surface covered by a turbulent boundary layer. The average heat-transfer coefficient was about 20% higher than would have been expected on a flat wall, but there was no evidence of large-scale streamwise structures. The wall image showed a structure of slowly undulating streaks having a lateral spacing of about 200 wall units and a streamwise length of about 1000 units. These values are very close to the values observed for the thermal structures on a flat wall, studied with the same technique. The spanwise variations in heat transfer were about  $\pm 15\%$  both on the flat wall and in the curved region. With one exception, the thermal streaky structure in the curved region was not distinguishably different from the pattern on the flat wall. The one difference noted was the intermittent appearance of large, isolated, "star-burst" patterns on the streak structure of the curved wall. During one of these events, the normally-parallel streaks in the wall image would suddenly diverge, forming a fan of rays pointing in the generally downstream direction. The divergence angles of the outermost rays would typically be on the order of  $30-45^\circ$ , top to bottom. The star-burst pattern would persist for several seconds, and then disappear by dissolving. The image did not convect downstream and did not appear to be washed out from its edges; it simply dissolved.

A 16 mm color motion picture was made of these events, in the flat and in the curved region, and transmitted to the Air Force\*. The details of the study are contained in a Thermosciences Division Report, HMT-35, by Simonich and Moffat, which has also been transmitted.

We believe that this study has established conclusively that the increase in the average heat-transfer coefficient in the concave region is not due to the direct action of streamwise vortices "scrubbing" the surface. As a corollary, then, we believe that the concave-wall heat-transfer problem can be handled by two-dimensional boundary-layer codes such as STAN5, given an appropriate model for the enhanced mixing.

To demonstrate the use of a two-dimensional program, a mixing-length model was installed in STAN5 and adjusted so that it successfully predicted the heat transfer.

The next phase of the experimental work on the heat-transfer portion of the program is awaiting the conclusion of the current round of hydrodynamic investigations. This delay in the heat-transfer work was anticipated in the proposal for 1984.

During 1984, a new Research Assistant was added to the group, Mr. Keith Hollingsworth. He replaces Mr. Kevin Stoll, who elected to seek employment in industry. Mr. Hollingsworth came to Stanford in September as a post-master's

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\*A copy was provided to AFOSR, Division of Mechanics, Bolling Air Force Base, in March of 1982.

student seeking a Ph.D. in the Thermosciences. He spent part of the first year familiarizing himself with the concave-wall heat-transfer problem, and learning to operate the experimental apparatus and instrumentation currently being used in the hydrodynamic study. It is expected that the transition from the hydrodynamic study of the heat-transfer study will be made more quickly and efficiently as a result of this training.

During the first half of 1984, the heat-transfer surfaces were taken out of storage and inspected for damage. The thermocouple probe was redesigned, and the new probe built.

During 1985, a major emphasis in the heat transfer portion of the work has been the investigation and development of image processing as an improved method to evaluate the images generated by the liquid crystal plates. In the past (Simonich and Moffat, 1982b), the color patterns produced by the liquid crystal heater packages were evaluated by human observation. This technique yields a measurement of the average heat transfer coefficient on the plate within an uncertainty bound of about 15%. It is our hope that computer processing of the color images will place the evaluation of the heat transfer coefficient at a point on the plate on a more quantitative and spatially precise level. Measurements of interest include: area average heat transfer coefficient, streak-dimension measurement, and the streak structure within the large scale sweeps or "splats" documented by Simonich and Moffat, 1982b.

The physical mechanisms by which liquid crystals generate colors in response to their temperature is well documented. The process leads to colors which are combinations of spectral colors and which can be described by a tristimulus decomposition using the three color-generating phosphors used in color television broadcasting. This means that a translation algorithm can be written which will calculate the dominant wavelength of a displayed color from the relative intensities of the Red, Blue, and Green electrical signals of an RGB color TV signal. The dominant wavelength can then be related to temperature through a calibration experiment. The result of this study was a convincing conclusion that color images of a liquid-crystal display can be interpreted digitally and can yield temperature distributions based on color (i.e., trichromatic signal interpretation), not simply on the intensity of a single hue (i.e., monochromatic signal interpretation). This will increase the precision of liquid-crystal measurements of heat-transfer rates using the basic technique already developed under this contract. This increase in precision may reveal aspects of the behavior within the streaky divergent regions which cannot be resolved by visual inspection.

A group of vendors was assembled at Stanford, each with his own equipment, and a demonstration system was assembled as a "plug-in" operation. Successful operation was demonstrated taking a liquid-crystal image directly from a color TV camera and also taking the image from a color TV recorder at low framing speed. Success is defined here as the acquisition of three arrays, one representing each of the R, G, and B signals and the subsequent regeneration of a sharp and steady image in each color alone, plus a combined (i.e., realistic) color image generated from the digital arrays. Standard color bars were included in the image field to assure that adjustments could be made to capture the perceived color and retain it through the processing.



During 1985, a system was purchased based on the results of that demonstration. The system is based on standard color video standard and the IBM PC computer and peripheral devices. We believe the system to be representative of the least expensive technique that can successfully demonstrate the concept. The system now includes the ability to produce a stable videotape freeze frame of a transient event, although the spatial resolution in this mode is one-half that obtained from normal tape speed. A list of the components and their function is included here. All components are off-the-shelf with no modification.

Panasonic tube-type color camera: generates a standard color video signal acceptable by the tape recorder and the color component decoder.

Panasonic 1/2" editing-grade video cassette recorder/player: for recording and playback of time-varying images.

For-A Digital Time Base Corrector: stabilized the freeze-frame signal from a video recorder, which without correction is not suitable for computer acquisition.

Lenco Color Component Decoder: decodes the color video signal from the camera, recorder, or time base corrector into three monochromatic signals containing the "red", "green", and "blue" information in the original signal.

Sony RGB color monitor: recombines the red, green, and blue signals to produce a color image.

Three Datacube digital image acquisition boards (internal to the IBM PC): The three boards digitize the red, green, and blue signals from the decoder and also convert these images back into analog signals for display on the monitor.

IBM PC computer with 21 MB fixed disk drive: used to control the acquisition boards, and to analyze and store the images.

Software has been developed for acquiring three-component color images and calculating the hue (color) and saturation (purity of color) of these images according to internationally accepted definitions of these terms. Calibration curves relating liquid crystal temperature to the calculated hue have been developed for sample liquid crystal surfaces. This calibration technique was demonstrated in the laboratory as part of a presentation at the Stanford Affiliates Conference in January, 1986. Software also have been developed for false-coloring monochromatic images to reveal subtle patterns in the image.

In order to test the equipment and develop and gain experience with the software, the system was applied to a number of suitable problems in our laboratory. Chief among these was software development to analyze images of "narrow-band" liquid crystals which produce a narrow band of color. These bands can be processed entirely as a monochromatic phenomena. Using this technique, the image processing system has been successfully used to identify the positions of isotherms throughout a two-dimensional field and reproduce the positions in standard graphical form. This application is quite similar to the concave channel problem of narrow sublayer streak detection.

Digital images have been acquired from photographs of various flows including: velocity streak measurement in particle seeded flows, enhancement of flow-visualization images, complete processing of narrow-band liquid crystal images of isotherms around a heated spot on a surface in mixed convection, and a full-color digitized image of heated water seeded with liquid crystal for temperature visualization. The last example listed may lead to a nonintrusive temperature field measurement technique for water flows. As a result of these exercises, we now believe digital image processing to be a working-level tool in the present research.

The imaging system is now being applied to the concave curvature channel. The existing liquid crystal plates used in the concave curvature channel (Simonich and Moffat, 1982b) have been tested for electrical and visual integrity in air and are now being installed in the channel for performance testing in water. The software for processing images from these plates is written and has been tested using photographs of liquid crystal events. Further development of the software is expected as particular questions concerning the application evolve.

The multiple-channel thermocouple probe discussed earlier will be used in conjunction with the liquid crystal plates to measure the temperature distribution vertically through the boundary layer. Calculations have been performed to determine the expected temperature distribution in a turbulent water boundary layer. These results have been used in a finite difference calculation to examine the response of the current thermocouple probe to the expected temperature distribution. The results will be used to determine proper placement of the probe within the boundary layer. The thermocouple probe will be used in conjunction with the liquid crystal plates during early summer. Approximately sixteen simultaneously sampled junctions will be used to provide a detailed documentation of the temperature distribution. Present plans involve the purchase of a standard sixteen channel analog-to-digital converter card for the project's IBM microcomputer.

The thermocouple probe will be used to measure conditionally sampled, ensemble averaged, temperature profiles. An appropriate trigger for the conditional sample may be the large scale and fairly infrequent heat transfer coefficient patterns of "splats" (Simonich and Mossat, 1982b) seen on the liquid crystal walls. Another trigger under consideration is the inflow/outflow regions seen in the dye visualizations of Barlow and Johnston (1985). Strategies for acquiring conditionally sampled temperature profile data will be tested as soon as the liquid crystal walls have been installed and are successfully operating.

### 3. Reports and Papers from Work under Part II of the Contract

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Barlow, R. S., and Johnston, J. P., "Velocity Spectra of Turbulent Boundary Layers on a Concave Surface," Fifth Symposium on Turbulent Shear Flows, Cornell University, Aug. 1985.

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- Jeans, A. H., and J. P. Johnston, "The Effects of Concave Curvature on Turbulent Boundary-Layer Structure," Structure of Complex Turbulent Shear Flow (R. Dumas and L. Fulachier, eds.), Springer-Verlag, 1983, pp. 89-99.
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- Simonich, J. C., and R. J. Moffat, "Visualization of the Heat Transfer through a Turbulent Boundary Layer on a Concave Wall," HMT-35, Mech. Engrg. Dept., Stanford Univ., Stanford, CA 94305, August 1982.
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